Prime Numbers, Prime Factors, Highest Common Factor and Lowest Common Multiple

A **prime number** is a positive integer that has exactly two distinct, positive factors.

Conversely, a **composite number** is a positive integer that has more than two distinct, positive factors.

One is neither a composite number nor a prime number.

Erasothenes' Sieve

Erasothenes was a Greek mathematician who was born around 276 BC in Cyrene in Libya. He studied in Athens and became a mathematician, astronomer, geographer and poet. He was the chief librarian at the Library at Alexandria. He is known as the father of geography, discovering that the Earth was a sphere and calculating its circumference. Mathematically, one of his interests was that of prime numbers and he came up with a method for determining prime numbers. This method is called Erasothenes' Sieve.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102

1. Find all the prime numbers in the table above by crossing out all those numbers that appear as multiples of the two to eleven times tables excluding the unitary members of each of the times tables.
2. Can you identify any type of pattern? (where do most of the prime numbers fall? Can you think of a mathematical way of describing their position?)

Fundamental Theorem of Arithmetic states that every positive integer greater than 1 can be represented as a ***unique*** product of its prime factors.

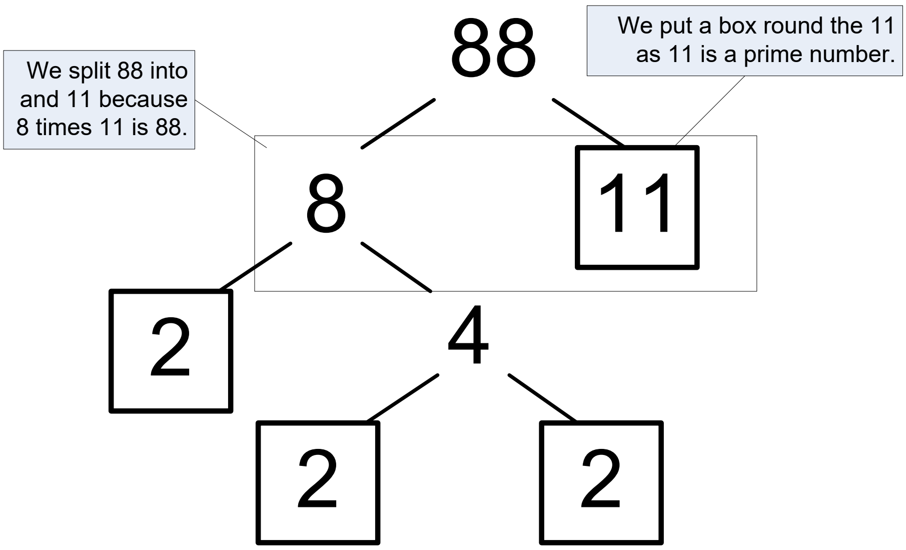
This was proved by Carl Frederick Gauss in 1801. Gauss (1777 – 1855) was a German mathematician and is regarded as one of the greatest mathematicians of all time.

To find the product of prime factors, we use a method called a **factor tree**.

$\boxed{7}$ $\boxed{13}$ $\boxed{2}$ $\boxed{29}$

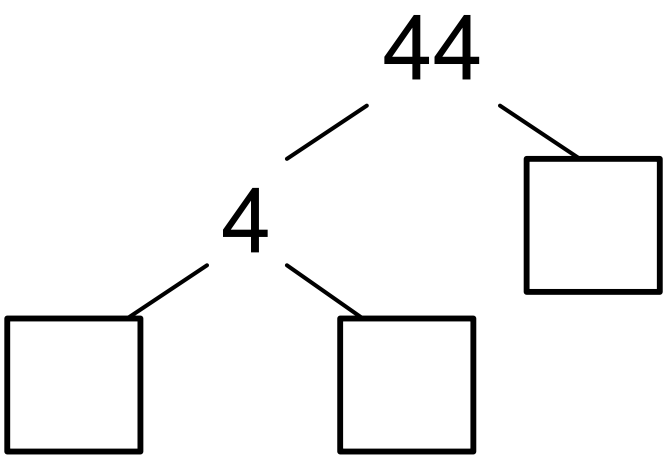
The numbers above are all prime numbers so in a factor tree, we put them in a box. Once we have come across a prime number, that branch of the factor tree stops at that point. Putting them in a box is to help us at a later stage.

Composite numbers don't immediately start as boxes. We have to do a little work to find out where the boxes go.



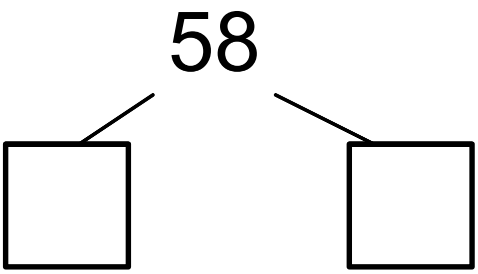
NOTE BENE: ***There are no ones in this prime factor tree***. One is not a prime number. This is one main place where people tend to go wrong.

3. Copy and complete the factor trees for the following numbers:



a.

b.



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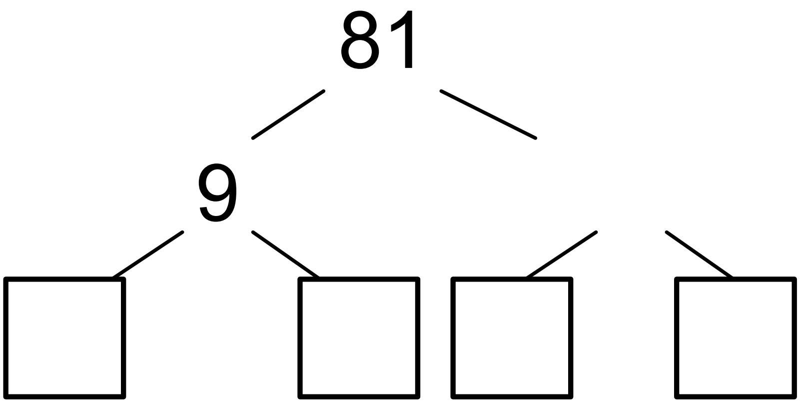
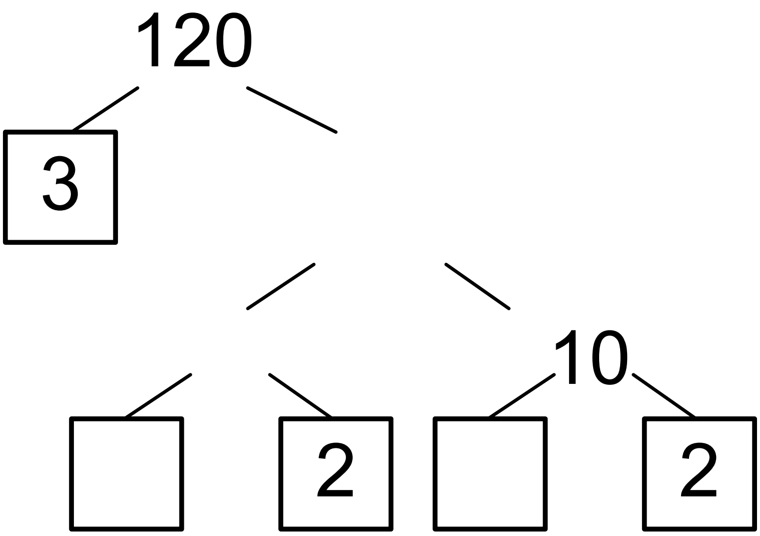


d.



With some numbers, there are lots of ways to find the solution, but you should always end up with the same answer, no matter which path you travel.

Once we have drawn a factor tree, expressing a number as a product of its prime factors is relatively simple.

In the example of 88, above, we have 2, 2, 2 and 11 in boxes. That is because these are all the prime numbers in the factor tree.

To express the number 88 as a product of its primes, we simply multiply these prime numbers together.

$$88 = 11 \times 2 \times 2 \times 2$$

However, we need to make this more concise. Firstly, we have to put the numbers into the numerical order, starting with the smallest number.

$$88 = 2 \times 2 \times 2 \times 11$$

Secondly, we group numbers together using powers.

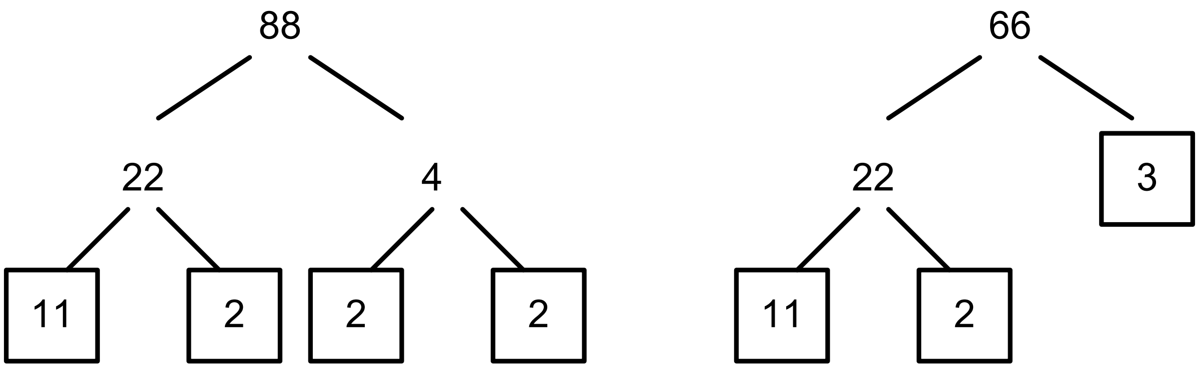
$$88 = 2^3 \times 11$$

4. Express each of the following numbers as a product of its prime factors.
    a. 240
    b. 56
    c. 140
    d. 31

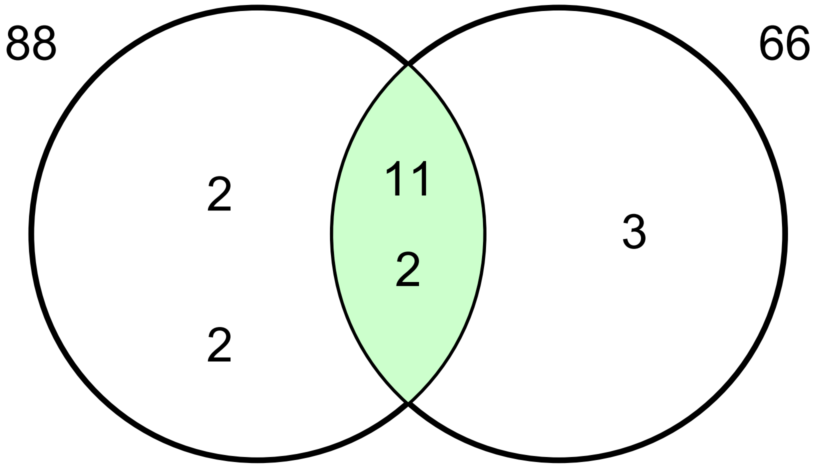
Highest Common Factors (also known as the Greatest common Divisor)

John Venn invented Venn diagrams in 1851 as a way of organising information. We use this method to find the highest common factor of two numbers.

To find the highest common factors of the numbers 66 and 88, we draw two factor trees.



We arrange the prime factors on a Venn diagram as shown below. Note that the numbers 2 and 11 are common factors so we write them once in the overlapping section of the Venn diagram and we only **write them once for both numbers**.



To complete the operation, we multiply the numbers in the intersection of the two circles together.

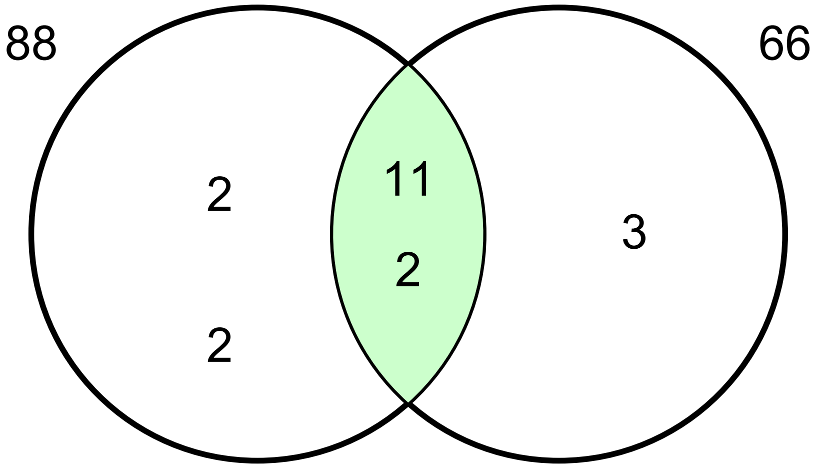
$$2 \times 11 = 22$$

So we can write HCF(66,88) = 22

5. Calculate the highest common factors of the following sets of numbers:
   a. Hcf(45, 80)
   b. Hcf (48, 900)
   c. Hcf (55, 220)
   d. Hcf (24, 90)

Lowest Common Multiple

To find the lowest common multiple, we follow the same process as for the highest common factor, apart from we multiply all the numbers in the Venn diagram together.



$$2 \times 2 \times 2 \times 3 \times 11 = 264$$

So we can write LCM(66,88) = 264

6. Find the lowest common multiple of the following pairs of numbers:
   a. (42, 80)
   b. (24, 60)
   c. (80, 300)
   d. (143, 374)

7. Three composite numbers are A, B and C.

   $A = 3^6 \times 5^3 \times 7$

   $B = 3^2 \times 5 \times 7$

   $C = 3^3 \times 5^2 \times 7^3$

   a What is the hcf (A,B,C) ?

   b What is the hcf(A,B)?

   c Put these into order of size of value: lcm (A,B), lcm (B,C), lcm (A,C)